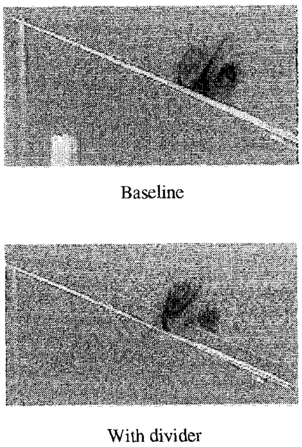

Fig. 4 Effect of D4b on vortex positions at station 3 at α = 25 deg and β = 10 deg.

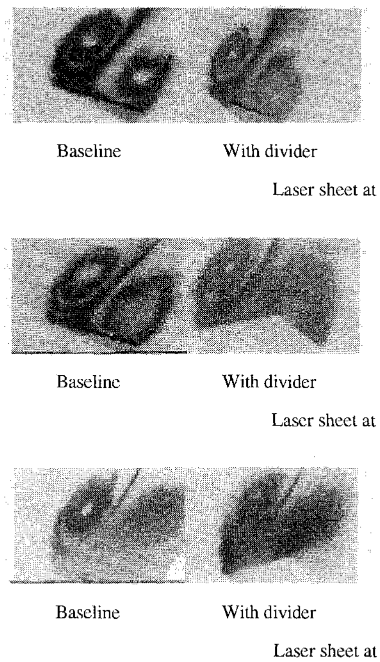

Fig. 5 Effect of D4b at α = 35 deg and β = 10 deg.

causes a small increase in C_l at small β compared with the baseline. Correspondingly, there is a small increase in the C_l gradient about zero β. An inspection of Fig. 2 shows a small increase in the wing rock amplitude at this condition.

Figure 4 shows the laser cross sections at α = 25 deg, and β = 10 deg. At this condition, the divider has a noticeable effect on vortex position asymmetry. The right (windward) vortex is closer to the surface compared with the baseline, whereas the left (leeward) vortex is farther away. The flow visualization result agrees with the rolling moment result where the rolling moment at sideslip is increased by the divider.

At α = 35 deg, shown in Fig. 3b, the rolling moment at sideslip is reduced by the divider. The laser cross sections at α = 35 deg and β = 10 deg, shown in Fig. 5, demonstrate the reason for the reduction in rolling moment. The breakdown of the windward vortex has propagated to a more forward position for the wing with the divider. The leeward vortex is displaced upward from the surface and seemingly weaker. An inspection of Fig. 2 shows a complete elimination of wing rock under this situation. Hence, the results indicate the divider decreases the static roll stability at high angles of attack, but increases the dynamic roll stability. Eventually, this leads to the suppression of wing rock.

IV. Summary and Conclusions

The effects of a flow divider placed on the leeward side of an 80-deg sharp-edged delta wing were studied. Effects of divider geometry, sizes, and placement were investigated. With a divider in the appropriate positions, wing rock is suppressed for angles of attack above 30 deg. At the lower range of the α, where the wing is naturally susceptible to wing rock, however, the divider can actually promote wing rock. These opposing effects on wing rock would prevent the fixed divider concept to be used for wing rock suppression.

Measurements indicate that the divider increases the rolling moment at sideslip condition for the angle-of-attack range where wing rock is promoted by the divider. Flow visualization shows that this is due to an increase in the vortex position asymmetry. The static stability is increased moderately, but the dynamic stability is reduced correspondingly. The divider therefore enhances the tendency for wing rock. At higher angles of attack where wing rock is suppressed by the divider, the rolling moment at sideslip is reduced by the divider. Flow visualization shows that this is due to the promotion of breakdown of the windward vortex. This in-turn leads to a decrease in the static stability and an increase in the damping. Wing rock is therefore suppressed.

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Integration of the Supersonic Kernel Function

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Nomenclature

B = $\sqrt{M^2 - 1}$
D_{rs} = velocity influence coefficient

Received Sept. 8, 1993; revision received Feb. 24, 1994; accepted for publication March 4, 1994.

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e = semispan of lifting surface element
I_0 = $\int_{s_1}^{s_2} \exp(-\omega_r s)/(r^2 + s^2)^{1/2}\, ds$, integral relating acceleration and velocity potential
K_1, K_2 = kernel functions $r(\partial I_0/\partial r)$ and $r^3\{(\partial/\partial r)[(1/r)/(\partial I_0/\partial r)]\}$
M = freestream Mach number
Q = singularity strength, defined in Eq. (1)
q = local acceleration potential doublet strength, normalized by $U^2/2$
R = $\sqrt{x^2 - B^2r^2}$
r = $\sqrt{y^2 + z^2}$
s_1 = $(x - MR)/B^2$
s_2 = $(x + MR)/B^2$
T_1, T_2 = direction cosine functions
U = freestream velocity
x, y, z = $x_0 - \xi, y_0 - \eta, z_0 - \zeta$
x_s = reference x coordinate of sending panel
x_0, y_0, z_0 = coordinates of receiving point in a sending panel coordinate system
ξ, η, ζ = coordinates of sending point in a sending panel coordinate system
ϕ = perturbation velocity potential, normalized by U
ω = angular frequency
ω_r = wave number $i\omega/U$

Introduction

PANEL methods for unsteady supersonic flow that use the acceleration potential formulation require the integration of the supersonic kernel functions. The expression for the nonplanar kernel function given by Harder and Rodden[1] has a nonintegrable singularity along the intersection of the forward Mach cone of the receiving point with the sending panel. Cunningham[2] showed that this singularity disappeared if the differentiation of the velocity potential was performed after integration over the sending panel. However, the intersection of the forward Mach cone of a receiving point with the leading or trailing edge of a sending panel still gives rise to a troublesome singularity. It is shown here how this singularity can be integrated using normal quadrature, thus allowing zero-order discontinuities in the pressure distribution.

Theory

The acceleration potential doublet is a convenient singularity to use in unsteady flow because there exists a simple relationship between the doublet strength and load on lifting surfaces, and the integration needed to calculate induced velocities and pressures is limited to the physical panel, i.e., excluding the wake. The expressions for the influence coefficients can be simplified by assuming that the doublet strength varies harmonically with ξ:

$$q(\xi, \eta) = Q \exp[-\omega_r(\xi - x_s)] \tag{1}$$

With this doublet strength distribution, the expression for the velocity potential induced by an acceleration potential doublet panel of unit strength ($Q = 1$) is

$$\phi = \frac{\exp[-\omega_r(x_0 - x_s)]}{8\pi} \int_{-e}^{+e} \int_{a+b\eta}^{c+d\eta} \frac{z_0}{r} \frac{\partial I_0}{\partial r}\, d\xi\, d\eta \tag{2}$$

where $\xi = a + b\eta$ and $\xi = c + d\eta$ define the panel leading and trailing edges, respectively. This expression is applicable if all of the sending panel lies within the forward Mach cone of the receiving point. In the case of a lifting surface panel being cut by the Mach cone, the induced potential is expressed as an integral from the leading edge to the Mach cone:

$$\phi = \frac{\exp[-\omega_r(x_0 - x_s)]}{8\pi} \left(\int_{\eta_1}^{\eta_2} \int_{a+b\eta}^{x_0 - Br} \frac{z_0}{r} \frac{\partial I_0}{\partial r}\, d\xi\, d\eta \right) \tag{3}$$

The spanwise integration limits can be either an edge of the panel or the intersection of the forward Mach cone of the receiving point with the leading edge of the sending panel. The expression for the influence coefficient is found by differentiating the expression for ϕ with respect to the normal of the receiving element. The kernel and direction cosine functions are introduced and the chordwise integration variable is changed to x

$$D_{rs} = \frac{\exp[-\omega_r(x_0 - x_s)]}{8\pi} \left\{ \int_{\eta_1}^{\eta_2} \frac{T_1}{r^2} \int_{Br}^{x_B} K_1(x, r)\, dx\, d\eta + \int_{\eta_1}^{\eta_2} \frac{T_2}{r^4} \left[\int_{Br}^{x_B} K_2(x, r)\, dx - BrK_1(Br, r) \right] d\eta \right\} \tag{4}$$

where $x_B = x_0 - (a + b\eta)$. Both K_1 and K_2 are singular at the lower limit of chordwise integration. The singularity of K_1 is integrable, but not the singularity of K_2. The singularity of K_1 in the integrand of the second spanwise integral cancels the chordwise integrated singularity of K_2 at $x = Br$. At spanwise integration limits defined by the intersection of the Mach cone with the leading edge, x_B goes to Br, and the integrand of the second spanwise integral goes to infinity. To resolve this singularity, K_1 is divided into a part K_1' which is zero at $x = Br$, and a part K_1'' which is singular at $x = Br$. (These are the same expressions as given in Ref. 1.)

$$K_1'(x, r, M, \omega_r) = -\int_{s_1}^{s_2} \frac{r^2 \exp(-\omega_r s)}{(r^2 + s^2)^{3/2}}\, ds \tag{5}$$

$$K_1''(x, r, M, \omega_r) = -\frac{Mr^2}{R} \left[\frac{\exp(-\omega_r s_2)}{(r^2 + s_2^2)^{1/2}} + \frac{\exp(-\omega_r s_1)}{(r^2 + s_1^2)^{1/2}} \right] \tag{6}$$

$K_1''(x_B, r)$ is added to the integrand of the second spanwise integral and the spanwise integral of $K_1''(x_B, r)$ subtracted separately:

$$D_{rs} = \frac{\exp[-\omega_r(x_0 - x_s)]}{8\pi} \left\{ \int_{\eta_1}^{\eta_2} \frac{T_1}{r^2} \int_{Br}^{x_B} K_1(x, r)\, dx\, d\eta + \int_{\eta_1}^{\eta_2} \frac{T_2}{r^4} \left[\int_{Br}^{x_B} K_2(x, r)\, dx + BrK_1''(x_B, r) - BrK_1''(Br, r) \right] d\eta - \int_{\eta_1}^{\eta_2} \frac{T_2}{r^4} BrK_1''(x_B, r)\, d\eta \right\} \tag{7}$$

The term $BrK_1''(x_B, r) - BrK_1''(Br, r)$ can be expressed as

$$BrK_1''(x_B, r) - BrK_1''(Br, r) = \int_{Br}^{x_B} Br \frac{\partial K_1''}{\partial x}\, dx \tag{8}$$

This yields an expression for D_{rs} in which the chordwise integrals have, after a change of integration variable to R, completely regular integrands:

$$dx = \frac{dR}{\left(\dfrac{\partial R}{\partial x}\right)} = \frac{dR}{\left(\dfrac{x}{R}\right)} = \frac{R}{x}\, dR \tag{9}$$

$$
\begin{aligned}
D_{rs} = {} & \frac{\exp[-\omega_r(x_0 - x_s)]}{8\pi} \\
& \times \left(\int_{\eta_1}^{\eta_2} \frac{T_1}{r^2} \int_0^{R_B} \frac{R}{x} K_1[x(R), r] \, \mathrm{d}R \, \mathrm{d}\eta \right. \\
& + \int_{\eta_1}^{\eta_2} \frac{T_2}{r^4} \int_0^{R_B} \frac{R}{x} \left\{ K_2[x(R), r] + Br \frac{\partial K_1''}{\partial x}[x(R), r] \right\} \mathrm{d}R \, \mathrm{d}\eta \\
& \left. - \int_{\eta_1}^{\eta_2} \frac{T_2}{r^4} BrK_1''(x_B, r) \, \mathrm{d}\eta \right) \qquad (10)
\end{aligned}
$$

where $R_B = \sqrt{x_B^2 - B^2 r^2}$. The first two spanwise integrals can be solved by normal quadrature or curve fitting followed by analytical integration as is done in the subsonic doublet lattice method.[3] The last spanwise integral can be written as

$$
\int_{\eta_1}^{\eta_2} \frac{T_2 B(R_B/r^2) K_1''(x_B, r)}{R_B r} \, \mathrm{d}\eta \qquad (11)
$$

where the numerator is a regular function. The problem with evaluating this integral is that r can become small at $\eta = y_0$ if z_0 is small, and R_B can become zero at the end points of the integration interval. In cases of interest these two conditions do not occur at the same value of η, therefore, the integration interval can be divided so that different techniques can be used depending on which one of r and R_B is smaller. Where r is smaller, a polynomial approximation is made to r times the integrand, and the resulting integral of a polynomial divided by r is solved analytically. The analytical integration is simplified by changing the integration variable to y:

$$
\begin{aligned}
& \int_{\eta_1}^{\eta_2} \frac{T_2 B(1/r^2) K_1''(x_B, r)}{r} \, \mathrm{d}\eta \\
& \approx \int_{y_1}^{y_2} \frac{b_4 y^4 + b_3 y^3 + b_2 y^2 + b_1 y + b_0}{\sqrt{y^2 + z^2}} \, \mathrm{d}y \qquad (12)
\end{aligned}
$$

where $y_1 = y_0 - \eta_2$ and $y_2 = y_0 - \eta_1$. Over those parts of the integration interval where R_B is smaller, the integral is cast in the form

$$
\int_{\eta_1}^{\eta_2} \frac{f(\eta)}{R_B} \, \mathrm{d}\eta \qquad (13)
$$

where $f(\eta) = T_2 B R_B K_1''(x_B, r)/r^3$ is a regular function. The integration variable is changed to u, given by

$$
u(\eta) = \int \frac{1}{R_B} \, \mathrm{d}\eta \qquad (14)
$$

$$
\mathrm{d}\eta = \frac{\mathrm{d}u}{\left(\dfrac{\mathrm{d}u}{\mathrm{d}\eta}\right)} = \frac{\mathrm{d}u}{\left(\dfrac{1}{R_B}\right)} = R_B \, \mathrm{d}u \qquad (15)
$$

The integral (13) can then be written as

$$
\int_{\eta_1}^{\eta_2} \frac{f(\eta)}{R_B} \, \mathrm{d}\eta = \int_{u(\eta_1)}^{u(\eta_2)} f[\eta(u)] \, \mathrm{d}u \qquad (16)
$$

which can be evaluated using normal quadrature. R_B can readily be expressed as the square root of a quadratic in η:

$$
R_B = \sqrt{a_2 \eta^2 + a_1 \eta + a_0} \qquad (17)
$$

For a panel leading edge defined by $\xi = a + b\eta$, the coefficients are

$$
\begin{aligned}
a_2 &= b^2 - B^2 \\
a_1 &= -2b(x_0 - a) + 2B^2 y_0 \qquad (18) \\
a_0 &= (x_0 - a)^2 - B^2(y_0^2 + z_0^2)
\end{aligned}
$$

The integral in Eq. (14) can be solved analytically with the result

$$
\begin{aligned}
u(\eta) &= \frac{2\sqrt{a_1 \eta + a_0}}{a_1} \qquad \text{if} \quad a_2 = 0 \\
u(\eta) &= \frac{1}{\sqrt{a_2}} \ln \left| 2\sqrt{a_2}\sqrt{a_2\eta^2 + a_1\eta + a_0} + 2a_2\eta + a_1 \right| \qquad \text{if} \quad a_2 > 0 \qquad (19) \\
u(\eta) &= \frac{-1}{\sqrt{-a_2}} \sin^{-1}\left(\frac{2a_2\eta + a_1}{\sqrt{a_1^2 - 4a_2 a_0}} \right) \qquad \text{if} \quad a_2 < 0
\end{aligned}
$$

The inverse relationship $\eta(u)$ is given by

$$
\begin{aligned}
\eta(u) &= \frac{\left(\dfrac{a_1 u}{2}\right)^2 - a_0}{a_1} \qquad \text{if} \quad a_2 = 0 \\
\eta(u) &= \frac{4a_2 a_0 - [a_1 + \exp(\sqrt{a_2} u)]^2}{4a_2 \exp(\sqrt{a_2} u)} \\
& \qquad \text{if} \quad a_2 > 0 \quad \text{and} \quad u < u_0 \\
\eta(u) &= \frac{[a_1 - \exp(\sqrt{a_2} u)]^2 - 4a_2 a_0}{4a_2 \exp(\sqrt{a_2} u)} \qquad (20) \\
& \qquad \text{if} \quad a_2 > 0 \quad \text{and} \quad u > u_0 \\
\eta(u) &= \frac{\sqrt{a_1^2 - 4a_2 a_0} \sin(-\sqrt{-a_2} u) - a_1}{2a_2} \\
& \qquad \text{if} \quad a_2 < 0
\end{aligned}
$$

where

$$
u_0 = \frac{\ln \sqrt{a_1^2 - 4a_2 a_0}}{\sqrt{a_2}} \qquad (21)
$$

Conclusions

It has been shown how the integrals resulting from a zero-order discontinuous pressure distribution can be arranged in such a way that they can be solved by either normal quadrature or curve fitting followed by analytical integration. This ability simplifies the panel method for unsteady supersonic flow and is essential to model the discontinuities that occur in reality, e.g., at supersonic leading or trailing edges and control surface hinge lines.

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